CHAPTER 4

Valuation Models and Metrics

Discounted Free Cash Flow and the Method of Multiples

In the two previous chapters we showed that the expected success of any business strategy can be evaluated based on whether it creates additional value for the owners of the firm. That said, the natural next questions are, how is created value measured, and, of the several valuation approaches that can be used, which is the most accurate? The IRS, for example, has sanctioned a number of valuation methods:

- *The asset approach.* This method first identifies a firm's tangible and intangible assets and values. The sum of these values is then equated to the value of the firm.
- *Income-based methods.* These methods project a firm's cash flow for valuation purposes over some period, discount these values to the present, and then sum these present values to obtain the value of the firm.
- *The method of multiples.* This method first identifies a set of firms that are comparable to the firm being valued. For each comparable firm, the ratio of its market price to revenue or earnings is calculated.[1] These ratios are averaged, and/or the median value is determined. The value of the target firm is then equal to the average or median revenue (earnings) multiple multiplied by the target firm's revenue (earnings).

As a theoretical matter, value should be independent of the valuation model used. As a practical matter, this is generally not the case. The reason is that the inputs that each method requires may not be consistent across valuation approaches, and hence a different answer emerges depending on which method is being used. For example, the income approach may indicate the firm is worth $1,000, and the method of multiples might indicate that firms like the target sell for three times revenue for a value of $1,200.

The reasons for this discrepancy are that the input values embedded in the comparable revenue multiple of 3 are different than the input values used in the income approach. Valuation analysts understand that the information required by each valuation model are not necessarily consistent and therefore accept the fact, with some limitation, that each valuation method will yield a different result. Alternatively, they also recognize that multiple valuations arising out of using different valuation approaches contain relevant and important information related to the underlying value of the firm. To cope with the inconsistencies and yet retain relevant information embedded in these different values, valuation analysts weight each value to create what is in essence an expected value of the target private firm. The weights represent an analyst's judgment about the "information quotient" embedded in each valuation approach, and to this extent, the weighting is strictly subjective.

This discussion raises an interesting question: Can one do better than simply use a subjective weighted average? Put differently, is there research that indicates, for example, which valuation model is likely to produce the smallest error? To this end, this chapter compares the two most commonly used valuation approaches—*discounted free cash flow* and the *method of multiples*. The latter approach is often used because it is simple to apply. The discounted cash flow approach is more complex because it requires information on a number of factors, including, the firm's true cash flow for valuation purposes, its cost of capital, its investment requirements, and the likely growth in revenue and profits. While these values are often difficult to calculate for public firms, they are far more difficult to estimate for private firms. Before addressing the issue of which valuation model is more accurate, this chapter first defines cash flow for valuation purposes and how to construct this concept from the financials of a private firm. It then goes on to derive the discounted free cash flow method, compares it to the method of multiples, and then reviews research that indicates that the valuation error is likely to be smaller using discounted free cash flow than using the method of multiples.

DEFINING CASH FLOW FOR VALUATION PURPOSES

To calculate a firm's cash flow for valuation purposes, we use the example of Tentex. Tentex, located in the Midwest United States, is a private firm operating in the packaging machinery industry, North American Industry Classification System (NAICS)-333993, or SIC 3565. This U.S. industry comprises establishments primarily engaged in manufacturing packaging machinery, such as wrapping, bottling, canning, and labeling machinery. This sector also includes the following activities:

- Bag opening, filling, and closing machines manufacturing.
- Bread wrapping machines manufacturing.
- Capping, sealing, and lidding packaging machinery manufacturing.

- Carton filling machinery manufacturing.
- Coding, dating, and imprinting packaging machinery manufacturing.
- Food packaging machinery manufacturing.
- Labeling (i.e., packaging) machinery manufacturing.
- Packaging machinery manufacturing.
- Testing, weighing, inspecting, and packaging machinery manufacturing.
- Thermoform, blister, and skin packaging machinery manufacturing.
- Wrapping (i.e., packaging) machinery manufacturing.

Tentex specializes in designing and manufacturing low- to moderate-volume machines that provide their customers with high-quality and cost-effective solutions through the innovative use of sensors, motion controls, and other technologies. Over the past several years, Tentex has developed a strong and a growing relationship with leading packaging companies. Tentex has become the outsourced designer and manufacturer of many of the machines that are either given or rented to customers for use in the customers' packing facilities. For example, a major Internet retailer is a client of one of Tentex's partners. The partner provides the retailer with Tentex machines to use with packaging materials purchased from the Tentex partner.

To arrive at cash flow for valuation purposes, several sets of adjustments to Tentex's reported income statement need to be made. To demonstrate these adjustments, we first start with Table 4.1, which shows Tentex's income statement for 2003. The column labeled Reported Value shows that Tentex reported no profit in 2003. However, after making a series of adjustments, Tentex had a pretax profit of $640,868. Total cash flow to owners and creditors before tax is the sum of reported pretax profit plus interest expense of $55,800, or a pretax total of $696,667.82. These adjustments are of two general types. The first is related to compensation of officers and other personnel related to the owners. The second relates to discretionary expenses, or expenses that were not necessary to the business.

Compensation of Officers and Employee Family Members and Friends

Reported compensation per owner/officer is $340,760. This compensation is divided into two components. The first component is a wage, and the second component is equivalent to a dividend each owner receives. To properly account for the true cost of labor, we need to determine the market wage (including benefits) that the firm would need to pay to obtain the same services each owner currently provides. Compensation less the market wage (including benefits) equals the dividend each owner receives.

Table 4.1 shows the benchmark wage for each owner. This benchmark

TABLE 4.1 Tentex Income Statement (2003) and Compensation and Discretionary Expense Benchmarks

Row	Concepts	Reported Value	Benchmark Value	Adjustment to Earnings	Adjusted Values
1	Gross receipts less returns and allowances	$3,562,556.00	—	—	$3,562,556.00
2	Cost of goods sold	$2,030,036.00	—	—	$2,030,036.00
3	Depreciation	$250,000.00			$250,000.00
4	Compensation of officers	$681,520.00	$258,574.00	($422,946.00)	$258,574.00
	Compensation of officer 1	$340,760.00	$129,287.00	($211,473.00)	$129,287.00
	Compensation of officer 2	$340,760.00	$129,287.00	($211,473.00)	$129,287.00
5	Salaries and wages	$350,000.00	$268,810.00	($81,190.00)	$268,810.00
	Bookkeeping clerk (wife)	$50,000	$28,650.00	($21,350.00)	$28,650.00
	Secretary (son)	$45,000	$26,390.00	($18,610.00)	$26,390.00
	Product promoter (brother)	$55,000	$25,360.00	($29,640.00)	$25,360.00
	Machinist (daughter)	$45,000	$33,410.00	($11,590.00)	$33,410.00
6	Repairs and maintenance	$1,800.00	—	—	$1,800.00
7	Rents	$18,400.00	—	—	$18,400.00
8	Interest	$55,800.00	—	—	$55,800.00
9	Other deductions	$175,000.00	$38,268.38	($136,731.62)	$38,268.38
	Travel expenses	$75,000	$22,045.10	($52,954.90)	$22,045.10
	■ Family vacation	$25,000	—	—	—
	■ Trip to Super Bowl	$10,000	—	—	—
	■ Family automobile	$35,000	—	—	—
	■ Fuel for family vehicles	$5,000	—	—	—
	Entertainment expenses	$45,000	$2,622.04	($42,377.96)	$2,622.04
	■ Company parties	$20,000	—	—	—
	■ Televisions	$15,000	—	—	—
	■ Season tickets to sports teams	$10,000	—	—	—

	Meals expenses	$50,000	$10,652.04	($39,347.96)	$10,652.04
	■ Family dinners	$35,000	—	—	—
	■ Sales dinners	$15,000	—	—	—
	Club expenses	$5,000	$2,949.20	($2,050.80)	$2,949.20
10	Taxable income	$0.00	—	—	$640,867.62

Benchmark Values for NAICS for Officer Compensation

Asset size	$100,000	$500,000	$1,000,000	$5,000,000	$25,000,000	$250,000,000
National	$59,870	$97,870	$133,818	$133,818	$182,970	$299,103
Illinois	$57,843	$94,556	$129,287	$129,287	$176,774	$288,974

Src: Axiom Valuation Solutions Compensation Database

Benchmark Values for NAICS for Discretionary Expenses

Expense	**Expense Benchmark (as percentage of total revenue)**
Travel expense	0.6188%
Entertainment expense	0.0736%
Meals expense	0.2990%
Club fee expense	0.0828%

Src: Axiom Valuation Solutions Discretionary Expense Database

Benchmark Value from Bureau of Labor Statistics for Worker Compensation

Occupation	**Value**
Bookkeeping clerk	$28,650.00
Secretary	$26,390.00
Product promoters	$25,360.00
Machinist	$33,410.00

Src: Bureau of Labor Statistics

is based on the firm's industry, asset size class, and location. Tentex's asset size, located in Illinois, is $5.0 million. The benchmark wage for each owner is $129,287. The difference between compensation paid per owner and this benchmark wage is $211,473. This dividend is added back to reported pretax profits. The total added back from this source is $422,946.

The same adjustment for owners is made for employee family members. It is not uncommon for owners to compensate family members in excess of what the firm would have to pay if it hired equivalently skilled third parties to do the same job. Like CEO wages, occupation wage levels vary by industry and geography. Based on data from the Bureau of Labor Statistics, Tentex pays family members close to twice their market wage. Based on these adjustments, pretax profits increase by $81,190.

Discretionary Expenses

Discretionary expenses are expenses incurred that are not necessary for the normal functioning of the business. Axiom Valuation Solutions has developed a database of discretionary expense percentages by industry. The raw data is from the Department of Commerce. Axiom has taken this information and has developed discretionary expense ratios by industry. The lower part of Table 4.1 shows the ratios applicable to Tentex. By multiplying each discretionary expense ratio by Tentex's total revenue, a discretionary expense benchmark is obtained. These benchmark values are then compared to actual discretionary expenses. If the actual expenditure exceeds its benchmark, costs are reduced by the amount of the difference, and pretax profits are correspondingly increased. In cases where the firm does not spend enough in a particular category, the expense level is raised and adjusted profits would decline. In some cases, the benchmark may not be appropriate. The analyst should be cautious about adjusting the reported benchmark in these cases. At a minimum, criteria should be developed based on hard data that offers guidance about the extent to which the reported benchmark should be adjusted.

In some cases, valuation analysts refer to industry benchmark values for officers' compensation published by the Risk Management Association (RMA) in its *Annual Statement Studies*[2] rather than following the method suggested here. Member banks provide survey information on about 15,000 firms each year across a large number of industries. RMA aggregates the data by industry and size and publishes what amounts to common-size income statements and balance sheets. For example, for most industries RMA publishes officer compensation as a percentage of revenue. When using the RMA data, the valuation analyst would multiply the RMA benchmark compensation ratio by the target firm's revenue to obtain a benchmark compensation value. The difference between this value and the reported compensation would be treated as excess compensation. If this difference is positive (negative), it would imply that officer's compensation should be adjusted downward (upward).

This approach tends to understate the portion of total compensation that should be treated as a dividend if the compensation data of the sample RMA firms includes bonuses. One could argue that a bonus is required to get the right people to run the firm and therefore it is a real cost of doing business. However, what happens if the firm performs poorly and funds are not available to pay the bonus? The answer is that no bonus is paid. A wage, by comparison, reflects an implicit contract between the firm and the employee. Therefore, the firm either pays the wage or terminates the employee. The bonus is a discretionary payment that is part of the return on capital and therefore like other payments to capital should be capitalized. This suggests that using the RMA officer's compensation benchmark to adjust reported officer compensation expense will likely result in the firm being undervalued.

Further Adjustments to Arrive at Cash Flow for Valuation Purposes

Once the financial statement of the private firm is unraveled, several additional adjustments need to be made to arrive at an accurate measure of firm profit, or net operating profit after tax (NOPAT). The lower section of Table 4.2 shows how NOPAT is calculated.

To move from pretax profit to NOPAT, the former must be reduced by taxes as shown on the income statement, which in this case amounts to 40 percent of pretax profits. This after-tax result is further reduced by the interest tax shield, or the tax savings that emerges because interest is a tax-deductible expense. This adjustment is made to reflect the true tax burden on the firm's assets, which is independent of how the assets are financed.[3]

Calculating Free Cash Flow to the Firm

Free cash flow to the firm is income available to shareholders and creditors after capital requirements are accounted for. It is equal to NOPAT plus interest expense, income available to shareholders and creditors, less the sum of net capital expenditure and the change in working capital adjusted for excess cash.[4] Table 4.3 shows the relationship between NOPAT and free cash flow to the firm.

Free cash flow to the firm is equal to $275,227. This is equal to NOPAT, $362,201, less the change in working capital, $69,783, less the change in net fixed capital, $17,192. Notice that depreciation is not added back in this calculation. The reason is that adding back depreciation to income available to shareholders and creditors is offset by subtracting

TABLE 4.2 Tentex Income Statement (2003) and Calculation of NOPAT

Row	Concepts	Reported Value	Benchmark Value	Adjustment to Earnings	Adjusted Values
1	Gross receipts less returns and allowances	$3,562,556.00	—	—	$3,562,556.00
2	Cost of goods sold	$2,030,036.00	—	—	$2,030,036.00
3	Depreciation	$250,000.00			$250,000.00
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	Product promoter (brother)	$55,000	$25,360.00	($29,640.00)	$25,360.00
	Machinist (daughter)	$45,000	$33,410.00	($11,590.00)	$33,410.00
6	Repairs and maintenance	$1,800.00	—	—	$1,800.00
7	Rents	$18,400.00	—	—	$18,400.00
8	Interest	$55,800.00	—	—	$55,800.00

9	Other deductions	$175,000.00	$38,268.38	($136,731.62)	$38,268.38
	Travel expenses	$75,000	$22,045.10	($52,954.90)	$22,045.10
	■ Family vacation	$25,000	—	—	—
	■ Trip to Super Bowl	$10,000	—	—	—
	■ Family automobile	$35,000	—	—	—
	■ Fuel for family vehicles	$5,000	—	—	—
	Entertainment expenses	$45,000	$2,622.04	($42,377.96)	$2,622.04
	■ Company parties	$20,000	—	—	—
	■ Televisions	$15,000	—	—	—
	■ Season tickets to sports teams	$10,000	—	—	—
	Meals expenses	$50,000	$10,652.04	($39,347.96)	$10,652.04
	■ Family dinners	$35,000	—	—	—
	■ Sales dinners	$15,000	—	—	—
	Club expenses	$5,000	$2,949.20	($2,050.80)	$2,949.20
10	Taxable income	$0.00	—	—	$640,867.62
11	Tax burden				
12	■ Taxes @ 40% (Row 10 × 0.4)				$256,347.05
13	■ Tax shield on interest (row 8 × 0.4)				$22,320.00
14	NOPAT				$362,200.57

TABLE 4.3 Tentex Balance Sheet and Calculation of Free Cash Flow

Row	Concepts Assets	2003	2002	Change: 2003/2002
1	Cash	$220,000	$187,000	
2	Cash required for operations	$71,251	$64,126	
3	Excess cash	$148,749	$122,874	
4	Accounts receivable	$356,256	$302,817	
5	Inventories	$890,639	$846,107	
6	Other current assets	$0	$0	
7	Total current assets	$1,686,895	$1,522,924	
8	Gross plant and equipment	$5,343,834	$5,076,642	
9	Accumulated depreciation	$3,730,729	$3,480,729	
10	Net fixed capital	$1,613,105	$1,595,914	
11	Total assets	$3,300,000	$3,118,838	
12	Liabilities and equity			
13	Short-term debt and current portion of long-term debt	$200,000	$190,000	

14	Accounts payable	$178,128		$160,315	
15	Accrued liabilities	$50,000		$42,500	
16	Total current liabilities	$428,128		$392,815	
17	Long-term debt	$490,000		$454,151	
18	Other long-term liabilities	$0		$90,000	
19	Deferred income taxes	$0			
20	Total shareholder equity	$2,381,872		$2,181,872	
21	Total liabilities and equity	$3,300,000		$3,118,838	
22	Working capital	$890,018	$0	$820,235	$69,783
23	Net fixed capital	$1,613,105	$0	$1,595,914	$17,192
24	Net capital requirements				$86,974
25	NOPAT	$362,201			
26	Free cash flow to the firm (row 25–row 24)	$275,227			

gross capital expenditures, which is defined as net capital expenditures plus depreciation.[5] Thus, depreciation is canceled out in the calculation of free cash flow to the firm.

Now that we know how to make the necessary adjustments to the financial statements of a private firm and in addition combine the adjusted income statement with the balance sheet to calculate free cash flow, we turn to the issue of valuing these cash flows. First, however, we review the cash flow valuation framework.

THE GENERAL VALUATION FRAMEWORK

The value of an ongoing business is related to the cash flow a buyer expects to receive from owning it. The buyer of the business expects the cash flows over time, and the size and timing of the cash flows, to be subject to a degree of uncertainty or risk. Therefore, for a business to be valued properly, the analyst needs to consider each of these factors. Finance theory tells us that if each of the valuation factors have been computed, then the value of a firm today should be equal to the sum of the present value of expected cash flow payments over the life of the asset, as shown in Equation 4.1.

$$V_0 = \frac{\hat{C}_1}{1+k} + \frac{\hat{C}_2}{(1+k)^2} + \ldots + \frac{\hat{C}_N}{(1+k)^N} \tag{4.1}$$

where V_0 = value

$\hat{C}_1 \ldots \hat{C}_N$ = expected value of free cash flow for future periods $1 - N$

k = the current discount rate

Predicting a firm's future cash flows is difficult to do with any degree of accuracy. Nevertheless, it may be possible to project the average growth rate in cash flow over an extended period of time with somewhat more accuracy. Equations 4.2 through 4.5 show the implications of imposing a constant cash flow growth on the general valuation model.

$$V_0 = \frac{C_0[1+\hat{g}]}{(1+k)} + \frac{C_0[1+\hat{g}]^2}{(1+k)^2} + \ldots + \frac{C_0[1+\hat{g}]^N}{(1+k)^N} \tag{4.2}$$

where $\hat{g}$ = the expected average annual growth rate of C and C_1 is equal to $C_0[1+\hat{g}]$

C_0 is the last cash payment received

If we define $(1+\hat{g})/(1+k)$ as λ, then V_0 is equal to $C_0\lambda[1+\lambda+\lambda^2+\ldots+\lambda^{N-1}]$.

If we assume that $(1 + \hat{g})$ always exceeds $(1 + k)$, the growth in C is greater than the discount rate k, then λ will be less than 1. If the life of the asset is long, N approaches infinity, then the term in brackets is the sum of a geometric series, which is equal to $1/(1 - \lambda)$.

$$V_0 = (C_0\lambda) \times \frac{1}{(1 - \lambda)} \qquad \text{or} \qquad V_0 = \frac{C_0[1 + \hat{g}]}{(k - \hat{g})} \tag{4.3}$$

This relationship is known as the *Gordon-Shapiro constant growth model.* Using this model, we now show that a firm's multiple is directly related to the present value of a firm's cash flow.

If we assume for a moment that the asset's value is equal to its market price, P_0, and the cash payment is defined as firm net income or traditional earnings, then the Gordon-Shapiro model yields the firm's price-earnings ratio.

$$\frac{P_0}{C_0} = \frac{[1 + \hat{g}]}{(k - \hat{g})} \tag{4.4}$$

The price-earnings multiple is an often-quoted valuation metric. To see how this multiple can be used to value the equity of a target firm, consider the following example. Let us assume that Firm A is a private firm whose shares have just been purchased for \$20 per share, and earnings per share is \$2. Hence, its price-earnings multiple is 10. Firm B is a private firm that is comparable to Firm A. If Firm B is currently earning \$1 per share, then the value of Firm B's equity, if it were publicly traded, would be \$10, or the per-share earnings of \$1 times the price-earnings multiple of 10. If we assume that Firm B has 1,000 shares outstanding and \$5,000 in debt, the value of the firm would be \$15,000 (\$10/share × 1,000 shares plus debt of \$5,000).

The price-earnings multiple is also directly related to the price-revenue multiple. To see this, assume that C_0 is equal to the current cash flow profit margin, m_0, multiplied by the most recent 12 months of revenue, R_0. Substituting $m_0 \times R_0$ for C_0 yields the revenue multiple P_0/R_0.

$$\frac{P_0}{R_0} = m_0 \times \frac{[1 + \hat{g}]}{(k - \hat{g})} \tag{4.5}$$

Note that the revenue multiple and the earnings multiple are a function of k, $\hat{g}$, and m_0. Thus two firms can be considered comparable if the values of these parameters are the same. Moreover, the value obtained for the target firm when applying the general valuation model directly, Equation 4.1, is likely to yield a different valuation result than the comparable method if the values k, $\hat{g}$, and m_0, implied by the general valuation model, are not consistent with the values of these parameters embedded in the multiples of the comparable firms. As a general rule, these parameters are rarely the same, and differences in value emerge because of this. We demonstrate this result in a subsequent section. However, first we introduce the nonconstant growth valuation model.

The Nonconstant Growth Valuation Model

The Gordon-Shapiro model can be made less restrictive by allowing cash flow growth rates over a finite time frame to vary from year to year and then assume that growth is constant from the end of the finite time frame forward. Imposing these assumptions on the general valuation equations yields Equation 4.6, the nonconstant growth model.

$$V_0 = \frac{\hat{C}_1}{1+k} + \frac{\hat{C}_2}{(1+k)^2} + \frac{\hat{C}_{n-1}}{(1+k)^n} + \ldots + \left(\frac{\hat{C}_{n-1}}{k-g}\right) \times \frac{1+g}{(1+k)^n} \quad (4.6)$$

The finite time frame between 1 and $n - 1$ is known as the *competitive advantage period*. It reflects a condition under which the firm earns a rate of return that exceeds its cost of capital. This condition is not expected to last forever, since earning monopoly rents will attract competitors that will bid down returns. As returns are bid lower, new investment opportunities with returns exceeding the cost of capital diminish. As a result, optimal use of internal funds requires that less of a firm's cash flow is used to finance new investment opportunities and more is returned to business owners in the form of dividends and distributions. As less of the firm's cash flow is used to finance new investment, the growth in future cash flows is lower as a result.

To see this consider the basic relationship between a firm's reinvestment rate, RR, rate of return on assets, ROA, and future growth in cash flows, g, as shown in Equation 4.7.

$$g_t = \text{ROA}_t \times \text{RR}_t \quad (4.7)$$

Now Equation 4.6 can be written as Equation 4.8:

$$V_0 = \text{CF}_0 \times \frac{(1 + \hat{\text{ROA}}_1 \times \text{RR}_1)}{(1+k)^1} = \frac{\hat{\text{CF}}_1}{(1+k)^1} + \hat{\text{CF}}_1 \times \frac{(1 + \hat{\text{ROA}}_2 \times \text{RR}_2)}{(1+k)^2}$$
$$= \frac{\hat{\text{CF}}_2}{(1+k)^2} + \ldots + \hat{\text{CF}}_{n-1} \times \frac{(1 + \hat{\text{ROA}}_n \times \text{RR}_n)}{(1+k)^n} = \frac{\left(\frac{\hat{\text{CF}}_n}{k-g}\right)}{(1+k)^n} \quad (4.8)$$
$$= V_0 = \text{CF}_0 \times [(1+\hat{g}_1)]/(1+k)^1 + [(1+\hat{g}_1) \times (1+\hat{g}_2)]/(1+k)^2$$
$$+ \ldots + \frac{[(1+\hat{g}_1) \times (1+\hat{g}_2) \times \ldots \times (1+\hat{g}_n)]}{(1+k)^n}$$

As the rate of return declines due to competitive pressures, the growth in cash flows will also decline. However, as long as ROA is greater than k, the retention rate should be large enough to fund investment requirements. In cases where investment requirements are less-than-expected after-tax cash flows, the retention rate is less than unity. When investment requirements exceed after-tax cash flows, then the firm needs outside funding in the form of new equity and/or debt. When competitive pressure results in a rate of return that equals the cost of capital, g will be zero because the retention will be zero. Put differently, reinvesting firm capital when the ROA equals k results in no additional value created by the investments made. When the long-run value of g is greater than zero, the firm has a sustainable competitive advantage, allowing it to earn rates of return that exceed the cost of capital in perpetuity.[6] Imposing competitive market conditions in Equation 4.8 implies that $\hat{g}_1 > \hat{g}_2 > \hat{g}_3 < \ldots > \hat{g}_n$, $\hat{g}_n = 0$ when k = ROA, and $RR_n = 0$. Thus, Equation 4.8 can be written as Equation 4.9.

$$V_0 = \hat{CF}_1/(1 + k)^1 + \hat{CF}_2/(1 + k)^2 + \ldots + [\hat{CF}_{n-1} \times (1 + \hat{g}_n)](k - \hat{g}_n)/(1 + k)^n \qquad (4.9)$$
$$\hat{g}_n = 0 \text{ if } k = \text{ROA}$$

Based on this discussion, one might ask: Is there an optimal value for g? While there is no optimal value per se, there is a plausible range. To start, the U.S. economy has a long-term growth rate of about 5 percent (3 percent real growth and 2 percent inflation). The long-term growth in firm cash flows should not be expected to grow significantly faster than the long-term growth potential of the U.S. economy. If this were assumed, it would imply that the firm would represent an increasing share of the total economy over time, and at some point in the future the firm would be equal in size to the total economy. This implication, of course, makes no sense, and hence the long-term value of g should reflect both the long-term competitive conditions facing the firm and the long-term growth potential of the total economy. This suggests that long-term growth rates in excess of the long-term growth of the economy are not sensible.

Valuing Tentex Using the Discounted Free Cash Flow Model

In this section we use the nonconstant growth model to value Tentex. The version of the model used combines Tentex's expected cash flow with its expected capital requirements to generate what is termed *free cash flow.* More precisely, free cash flow is defined as NOPAT less the change in working capital and net capital expenditures. Table 4.4 shows the inputs used in the Tentex valuation. Table 4.5 shows the Tentex valuation and the various components that make it up.

Note that Tentex revenue is expected to grow at 7 percent a year for each of the next four years and then to slow as expansion opportunities diminish. Revenue growth is a function of two factors. They are the expected revenue growth of the packaging equipment industry and Tentex management's ability to execute its strategy. Tentex is not a national player but does service a large market area centered in the Midwest. Thus, expected Tentex's revenue growth reflects both the expected national growth of the industry and the expected nominal economic growth of Tentex's service territory.[7]

Growth in taxable income reflects management's intention to consistently increase the efficiency of its manufacturing and distribution operations. Thus, growth in taxable income is expected to exceed growth in revenue. Tentex has debt outstanding of $679,039, which will increase as it finances part of its future capital additions with debt. Interest expense will remain constant, however, since management will adjust maturities of new debt in response to expected rate changes. As rates rise, management will seek out lower rates by issuing shorter-dated debt, and it will do the opposite when rates fall. Net fixed and working capital increase at the same rate as revenue, as suggested by the multiplier theory of investment.[8] Changes in net fixed capital and working capital are equivalent to net capital expenditures and change in working capital, respectively. These values are subtracted from cash flow to shareholders and creditors to obtain free cash flow. Tentex's cost of capital is 12 percent. In Chapter 5, we show how the cost of capital is calculated. For the moment, think of this rate as a blend of

TABLE 4.4 Data Inputs Used to Value Tentex

Inputs	Values	Source
Depreciation and amortization growth rate	3.00%	Growth in revenue
Net fixed assets:		
Starting value	$1,613,105.00	Balance sheet
Revenue growth	7.00%	Based on industry growth factors
Net working capital:		
Starting value	$890,018.00	Balance sheet
Cost of capital	12.00%	Calculated
ROA in perpetuity	15.00%	Based on analysis of long-term competitive factors
Retention rate	20.00%	Based on investments that have returns in excess of 12%
Long-term growth	3.00%	Based on analysis of long-term competitive factors
Tax rate	40.00%	Statutory rate
Initial debt level	$490,000.00	Balance sheet

TABLE 4.5 Valuing Tentex Using the Discounted Free Cash Flow Model

Time Period	0	1	2	3	4	5	6	Value in Perpetuity
Revenue	$3,562,556	$3,811,935	$4,078,770	$4,364,284	$4,669,784	$4,809,878	$4,954,174	
Revenue growth		7.00%	7.00%	7.00%	7.00%	3.00%	3.00%	
Taxable income growth: Competitive advantage period		21%	10.00%	15.00%	10.00%	6.00%	5.00%	
Taxable income	$640,868	$774,051	$851,456	$979,175	$1,077,092	$1,141,718	$1,198,804	
Interest expense	$0	$55,800	$55,800	$55,800	$55,800	$55,800	$55,800	
Tax @ 40%	$256,347	$309,621	$340,583	$391,670	$430,837	$456,687	$479,522	
Tax shield on interest	$0	$22,320	$22,320	$22,320	$22,320	$22,320	$22,320	
Tax burden	$256,347	$331,941	$362,903	$413,990	$453,157	$479,007	$501,842	
NOPAT	$384,521	$442,111	$488,554	$565,185	$623,935	$662,711	$696,962	
Growth in NOPAT		15%	11%	16%	10%	6%	5%	
Cash flow to owners and creditors after tax		$442,111	$488,554	$565,185	$623,936	$662,711	$696,962	
Net fixed capital	$1,613,105	$1,726,022	$1,846,844	$1,976,123	$2,114,452	$2,177,885	$2,243,222	
Net capital expenditure		$112,917	$120,822	$129,279	$138,329	$63,434	$65,337	
Net working capital	$890,018	$1,074,979	$1,182,477	$1,359,848	$1,495,833	$1,585,583	$1,664,862	
Change in working capital		$184,961	$107,498	$177,372	$135,985	$89,750	$79,279	
Free cash flow		$144,233	$260,235	$258,535	$349,622	$509,527	$552,347	$7,976,347
Present value		$128,779	$207,457	$184,020	$222,191	$289,119	$279,836	$4,041,066
Sum present value	$5,352,469							
Debt level*	$679,039							
Tentex equity	$4,673,430							
Liquidity discount rate	20.00%							
Discount due to liquidity	$934,686							
Equity less liquidity discount	$3,738,744							
Value of debt	$679,039							
Value of Tentex	$4,417,783							

*Market value of debt at the valuation date.

Tentex's after-tax equity and debt costs. As new capital additions are made, these assets are financed on an after-tax basis at 12 percent.

By discounting the expected free cash flows to the present at Tentex's cost of capital, the value of these cash flows is $5,352,469. The value of Tentex equity is this total less $679,039, or $4,673,430. One final adjustment needs to be made to this value. Remember that Tentex is a private firm, so its equity does not trade in a liquid market. Since the Tentex cost of capital was developed from factors that apply to firms whose equity trades in a liquid market, an adjustment must be made for the lack of liquidity, or marketability, of its equity.[9] In Chapter 6, we address this issue in much more detail, but for now we simply apply a discount of 20 percent for lack of marketability. This reduces the value of equity to $3,738,744. Adding back the initial value of debt yields a total value for Tentex of $4,417,783.

What Multiples Tell Us about the Value of Tentex

An important reason often given for using a multiples approach in conjunction with discounted free cash flow is to assess whether the latter yields a value consistent with market prices. In the analysis that follows, the equity multiple is used to calculate Tentex's equity value. The market value of debt is added to this value to obtain total firm value, which can also be calculated using the free-cash-flow-to-the-firm approach. The problem with using equity multiples is that it assumes that the multiples being used are directly applicable to the target firm. Let us explore whether this is indeed the case for Tentex.

Our search indicated that the comparable firms were all public companies. These firms operated in the same industry as Tentex, but each firm operated in slightly different industry segments. Nevertheless, Tentex and these comparable firms were generally impacted by the same economic and industry forces, and hence in this respect they offered useful valuation benchmarks. The data used in this analysis is shown in Table 4.6.

The comparable analysis we are about to undertake uses only the price-to-sales multiple as the valuation metric. While price-to-earnings (net income) multiples are often used as valuation metrics, these are characterized by a great deal of variability relative to the more stable revenue multiple. There are two reasons for this. First, sales are less subject to accounting distortions than earnings. Second, current earnings are far more variable than equity values, often leading to large year-to-year swings in the earnings multiple. Revenue, on the other hand, is generally far less variable than earnings, contributing to relatively less volatility in the revenue multiple. For these reasons, the revenue multiple is likely to be a better value metric to use as a standard of comparison than is the discounted free cash flow valuation.[10]

To place the comparable firms on a more equal footing relative to Tentex, we proceeded in two steps. In step 1, the value of *g* for each comparable

TABLE 4.6 Financial Information of Peer Firms

Company Name	Unlevered Beta	Reported Debt-to Equity-Ratio	Size Premium	Actual Levered Cost of Equity Capital	Cost of Equity: 90/10	Net Income Profit Margin	P/S Ratio	Implied *g*: Gordon Model	Adjusted Implied *g*	Estimated P/S
Cuno Inc.	0.4199	0.02	0.43%	8.40%	8.57%	9.30%	2.753	4.86%	3.00%	2.50
Esco Technologies, Inc.	0.4157	0.02	0.43%	8.37%	8.54%	6.74%	1.613	4.02%	2.00%	1.60
Flow International Corp.	0.4365	2.26	3.16%	15.60%	11.43%	–48.58%	0.272	–246.78%	NM†	NM
Nordson Corp.	0.3974	0.19	0.34%	8.45%	8.13%	5.27%	1.949	5.59%	4.60%	2.11
Pall Corp.	0.3846	0.18	0.34%	8.33%	8.20%	6.40%	1.857	4.72%	3.00%	1.85
Peerless Manufacturing Co.	0.4512	0.00	4.21%	12.38%	12.60%	–0.55%	0.458	13.74%	NM	NM
Taylor Devices, Inc.	0.4617	0.85	4.21%	14.21%	12.68%	2.53%	0.485	8.55%	8.00%	0.66
TB Woods Corp.	0.4512	0.65	4.21%	13.68%	12.60%	–0.37%	0.407	14.73%	NM	NM
Average*				11.18%	10.35%	6.05%	1.22	5.55%	4.12%	1.75
Tentex				15.00%	15.00%	10.79%		3.00%	3.00%	1.36‡

*Average based on positive values only. †Not meaningful. ‡Discounted cash flow multiple.

firm was determined and compared to the 3 percent used in the discounted free cash flow model. Each firm's *g* was solved for by assuming its price-sales ratio was established according to the Gordon-Shapiro model. This is termed the implied *g*. Then each firm's cost of equity capital was substituted into the Gordon-Shapiro model and each firm's implied *g* was solved for. As Table 4.6 indicates, the implied *g* for each firm was greater than 3 percent, with the average being almost twice as large, or 5.55 percent.

However, these two rates may not be fully consistent. The reason is that the differential could be a product of each firm having high near-term growth rates that are similar to Tentex, and yet the Gordon-Shapiro model forces these values to be averaged with the true long-term growth rate to produce an implied *g* that is greater than 3 percent.

To test this possibility, Equation 4.10 was solved for each comparable firm's adjusted implied *g*, designated as $\hat{g}_n$. The values of $g_1 \ldots g_6$ are equal to those used in the Tentex discounted free cash flow valuation.

$$\begin{aligned} V_0/R_0 = m_0 \times [&(1+\hat{g}_1)/(1+k)^1 + \ldots + (1+\hat{g}_1)\times(1+\hat{g}_2) \\ &\times \ldots \times (1+\hat{g}_6)/(1+k)^6 + (1+\hat{g}_1)\times(1+\hat{g}_2) \\ &\times \ldots \times (1+\hat{g}_6)\times(1+\hat{g}_n)/(k-\hat{g}_n)/(1+k)^6] \end{aligned} \tag{4.10}$$

$$V_0/R_0 = \text{revenue multiple}$$

The results of this analysis, although not shown separately, indicate that the average value of $\hat{g}_n$ is 4.12 percent. In step 2, a new cost of capital was calculated for each firm based on Tentex's target capital structure—90 percent equity and 10 percent debt.[11] Using the adjusted implied *g*, $\hat{g}_n$, and each firm's new equity cost of capital, each firm's estimated price-to-sales ratio was calculated assuming the Gordon-Shapiro model was operative. These values are shown in the column headed Estimated P/S in Table 4.6. The average of these values is 1.75, which is the average comparable multiple adjusted for Tentex's capital structure and each comparable firm's expected long-term growth in earnings. By comparison, the discounted cash flow equity multiple before an adjustment for marketability is 1.36.[12] This difference emerges because the values of the key parameters that determine the revenue multiple profit margin, near- and long-term earnings growth rates and the equity cost of capital, are significantly different for Tentex relative to the set of comparable firms. Nevertheless the comparable analysis did indicate that the long-term earnings growth may be greater than the 3 percent assumed for Tentex. To the extent that Tentex has potential for long-term earnings to grow at 4 percent instead of 3 percent, this should be factored into the valuation. We recalculated Tentex's discounted cash flow value using the 4 percent long-term growth rate. This raised the revenue multiple to 1.51, and the value of Tentex to $4,806,582, compared to the initial estimate of $4,673,430.

How does one reconcile these values? One way is to ask the question, what is the probability that Tentex's long-term growth will be 4 percent instead of 3 percent? Guidance for this determination should come from the valuation analyst's understanding of the nature of the business and the basis for the firm's competitive advantage. If we assume for the moment that this guidance suggested a 20 percent chance of achieving the 4 percent growth rate, and an 80 percent chance of a 3 percent growth rate, then Tentex's value would be equal to the weighted average of the two values, where the weights are the respective probabilities.

$$\text{Tentex equity value} = 0.8 \times (\$4{,}673{,}430) + 0.2(\$4{,}806{,}582) = \$4{,}700{,}060$$

This analysis suggests that simply using the average or median of comparable multiples when the values of the key parameters of these firms do not match the values of these parameters for the target firm will result in firm values that are subject to a great deal of error. Since the long-term growth rate is an important determinant of firm value, comparable multiples can be used to gauge whether the long-term growth rate assumed for the target firm is consistent with investor expectations. This growth rate can then be used to recalculate the value of the firm using the discounted free cash flow approach. Finally, a weighted average of the two discounted free cash flow estimates can be calculated to determine the final value of the firm.

DISCOUNTED CASH FLOW OR THE METHOD OF MULTIPLES: WHICH IS THE BEST VALUATION APPROACH?

Discounted cash flow approaches are used routinely by Wall Street and buy-side analysts to value firms they view as potential investment candidates. Despite the acceptance of the discounted cash flow approach by the professional investment community, there is less support for its use by the valuation community that specializes in valuing private firms. A reason often given for this reluctance is that its use requires growth in revenue and earnings to be projected forward, and hence there is a great deal of uncertainty that surrounds these projections and the estimated value of the firm. By comparison, it appears on first glance that the method of multiples does not require the analyst to make any projections, but merely to carry out the required multiplication to calculate the value of the firm. However, as the preceding analysis indicates, this view is not correct. If the method of multiples is used without any adjustments to the parameters that determine its value, the valuation analyst is accepting projections that are embedded in the multiple being used. If these projections are inconsistent with the target firm's potential performance, the value placed on the target firm will be incorrect. Hence, both valuation metrics are subject to forecasting error. The question is which method is likely to be the most accurate? We now turn to the answer to this question.

Steven Kaplan and Robert Ruback performed an exhaustive study of this issue. The authors state:

> *Surprisingly, there is remarkably little empirical evidence on whether the discounted cash flow method or the comparable methods provide reliable estimates of market value, let alone which of the two methods provides better estimates. To provide such evidence, we recently completed a study of 51 highly leveraged transactions designed to test the reliability of the two different valuation methods. We chose to focus on HLTs [highly leveraged transactions]—management buyouts (MBOs) and leveraged recapitalizations—because participants in those transactions were required to release detailed cash flow projections. We used this information to compare prices paid in the 51 HLTs both to discounted values of their corresponding cash flow forecasts and to the values predicted by the more conventional, comparable-based approaches. We also repeated our analysis for a smaller sample of initial public offerings (IPOs), and obtained similar results.*[13]

The basic results of the Kaplan and Ruback study are shown in Table 4.7.

The researchers developed several estimates of value by combining projected cash flows that were available from various SEC filings with several estimates of the cost of capital developed using the capital asset pricing model, or CAPM (CAPM-based valuation methods). Beta, the centerpiece of the CAPM and a measure of systematic risk, was measured in three different ways. In Table 4.7, the median value of each beta type is in the Asset beta row. The Firm Beta column was measured using firm stock return information. The Industry Beta column was developed by aggregating firms into industries and then using industry return data to measure beta. The Market Beta column was estimated using return data on an aggregate market index.

The researchers defined comparable firms in three ways. The comparable firm method used a multiple calculated from the trading values of firms in the same industry. The comparable transaction method used a multiple from companies that were involved in similar transactions. The comparable industry transaction method used a multiple from companies that were both in the same industry and involved in a comparable transaction. Columns A through F show the errors associated with each valuation method. The firm beta–based

TABLE 4.7 Comparison of Free Cash Flow Valuation to the Method of Multiples

	CAPM-Based Valuation Methods			Comparable Valuation Methods		
	(A) Firm Beta	(B) Industry Beta	(C) Market Beta	(D) Comparable Company	(E) Comparable Transaction	(F) Comparable Industry Transaction ($N = 38$)
Panel A: Summary statistics for valuation errors						
1. Median	6.00%	6.20%	2.50%	–18.10%	5.90%	–0.10%
2. Mean	8.00%	7.10%	3.10%	–16.60%	0.30%	–0.70%
3. Standard deviation	28.10%	22.60%	22.60%	25.40%	22.30%	28.70%
4. Interquartile range	31.30%	23.00%	27.30%	41.90%	32.30%	23.70%
5. Asset beta (median)	0.81	0.84	0.91			
Panel B: Performance measures for valuation errors						
1. Pct. within 15%	47.10%	62.70%	58.80%	37.30%	47.10%	57.90%
2. Mean absolute error	21.10%	18.10%	16.70%	24.70%	18.10%	20.50%
Mean squarred error	8.40%	6.70%	5.10%	9.10%	4.90%	8.00%

discounted cash flow method had a median error of 6 percent. This means that the median estimated transaction value was 6 percent greater than the actual transaction price. The median errors for the industry and market betas were 6.2 percent and 2.5 percent, respectively. In comparison, the comparable company multiple had a median error of –18 percent, while the comparable transaction multiple had an error rate that was equivalent to the firm and industry beta discounted cash flow results. When the multiple reflects the industry and the transaction of the target firm, the error is close to zero.

While the multiple approaches seem to produce error rates similar to the discounted cash flow approach, further examination suggests that this is not the case. Column B in Table 4.7 indicates the percentage of transactions that were within 15 percent of the actual transaction price. The discounted cash flow method had a greater number of estimated transaction values within 15 percent of the actual transaction price than do the comparable approaches. The mean square error of the discounted cash flow approach is generally smaller than the mean square error for the comparable methods. The results taken together support the conclusion that the discounted cash flow is more accurate than a multiple approach, although the errors are likely to be lower if the methods are used together. Kaplan and Ruback conclude:

> *Although some of the "comparable" or multiple methods performed as well on an average basis, the DCF methods were more reliable in the sense that the DCF estimates were "clustered" more tightly around actual values (in statistical language, the DCF "errors" exhibited greater "central tendency"). Nevertheless, we also found that the most reliable estimates were those obtained by using the DCF and the comparable methods together.*[14]

SUMMARY

Several critical adjustments need to be made to the reported financial statements of private firms in order to properly calculate cash flow for valuation purposes. These include officer compensation and discretionary expense adjustments. If the firm has debt on the balance sheet, then the firm's reported tax burden must be increased by the tax shield on interest. NOPAT is calculated as taxable income less tax paid less the interest tax shield. Free cash flow equals NOPAT less change in working capital and net capital expenditures. Discounting expected free cash flow yields the value of the firm. Alternatively, the method of multiples can be used to value a private firm. Research suggests that the discounted free cash flow method is a more accurate valuation approach.